

AN ELSCM METHOD FOR UNSTEADY-STATE ADVECTION-DIFFUSION EQUATIONS

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We developed a nonconventional Eulerian-Lagrangian single-node collocation method (ELSCM) with piecewise-cubic Hermite polynomials as basis functions for the numerical simulation to unsteady-state advection-diffusion transport partial differential equations.

Based on conventional collocation methods which attempt to minimize the residual in the variational formulation or the weighted residual methods for the governing differential equations by forcing the residual to be zero at a finite number of discrete or collocation points within the physical domain, Wu and Pinder [2] introduced a single-node strategy to simplify the standard collocation methods which use Hermite polynomials as basis functions, primarily by replacing lower order space derivatives in the trial functions by corresponding upstream-weighted, finite-difference quotient approximations. This treatment reduces the total number of unknowns.

To avoid of the time truncation errors that cause numerical diffusion and suffer from the Courant-Friedrich-Levy (CFL) restriction, we applied Eulerian-Lagrangian methods which follow the movement of information or particles as well as their interactions along the characteristics. Because solutions of transient advection-diffusion equations are much smoother along the characteristics than they are in the time direction, these methods have greatly reduced truncation errors comparing with the Eulerian treatments, have generated accurate numerical solutions even if large time steps are employed, and have relaxed the CFL restriction [1].

Wu and collaborators combined the nonconventional collocation method with Eulerian-Lagrangian concepts [3]. The ELSCM method generates very accurate and satisfactory numerical solutions for one dimensional transport equations, despite that the method used large time steps and greatly reduced numbers of unknown variables. It has been extended to solve highly unsteady-state transport problems in multidimensional spaces. Numerical experiments in one-, two-, and three-dimensional spaces are presented to show the strong potential of this method.

References

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